Adapting Cryptographic Tools to a Post-Quantum Context

Or how to improve HQC

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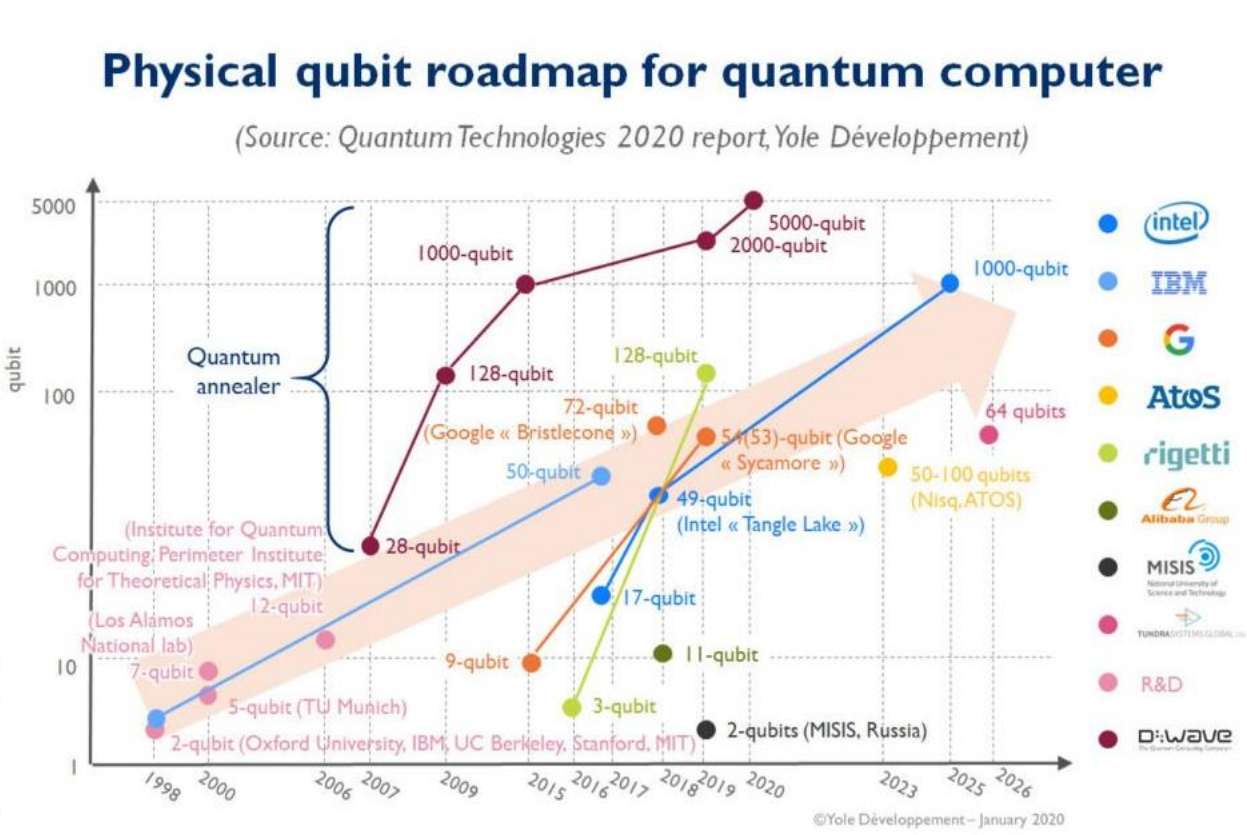
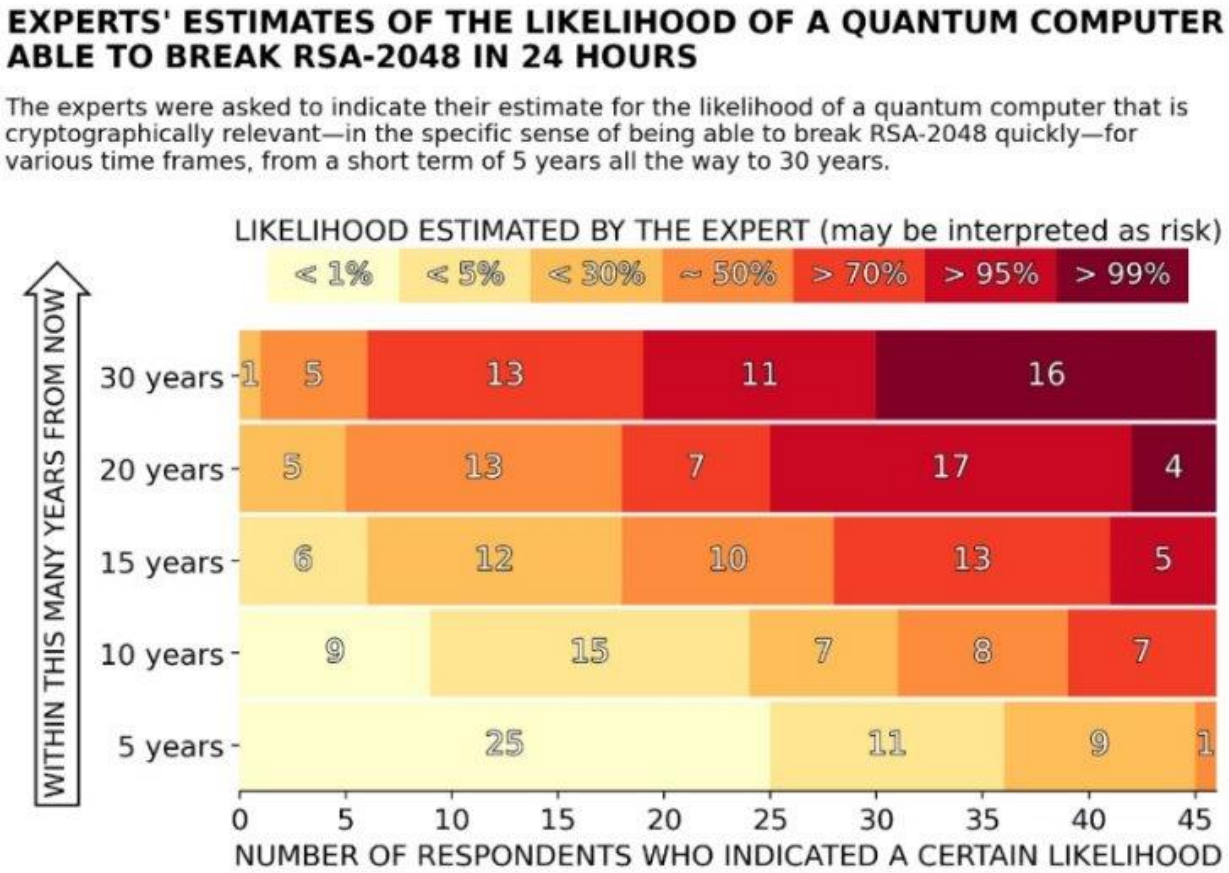
Why post-quantum cryptography ?





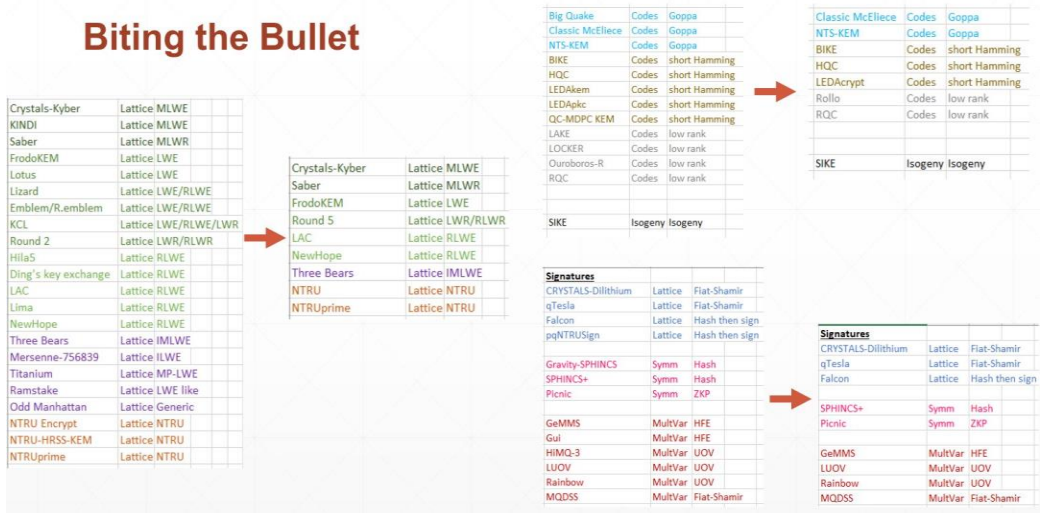
When post-quantum cryptography ?

2016 : Call to standardization (69 submissions)

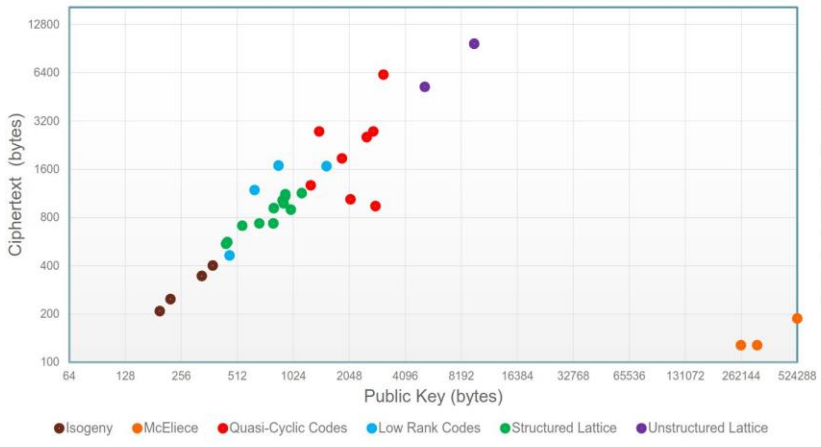
4 Rounds of selections (mathematical and performance criteria)

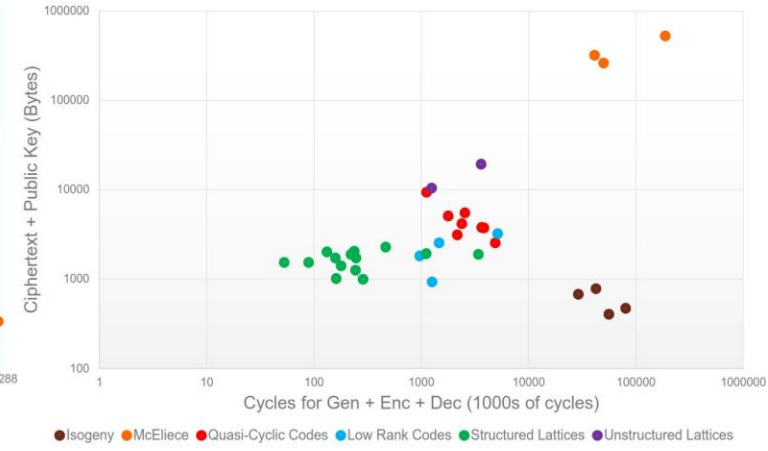
2025 : Two standardizations: Kyber and HQC

2035 : Classical cryptography disallowed by NIST and ANSSI



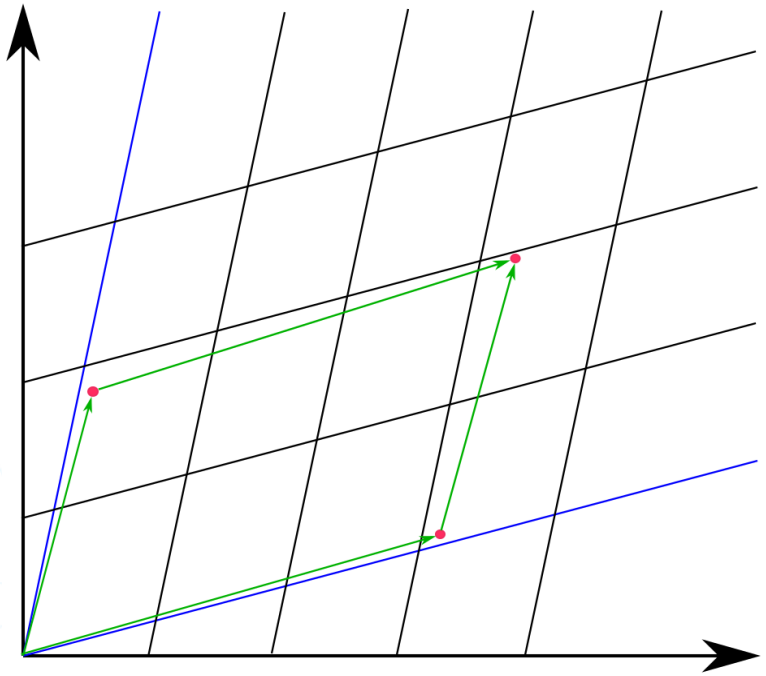
Key Establishment Scheme	Parameters	Transition
Finite Field DH and MQV [SP80056A]	112 bits of security strength	***Deprecated*** after 2030
		Disallowed after 2035
	≥ 128 bits of security strength	***Disallowed*** after 2035
Elliptic Curve DH and MQC [SP80056A]	112 bits of security strength	***Deprecated*** after 2030
		Disallowed after 2035
	≥ 128 bits of security strength	***Disallowed*** after 2035
RSA [SP80056B]	112 bits of security strength	***Deprecated*** after 2030
		Disallowed after 2035
	≥ 128 bits of security strength	***Disallowed*** after 2035





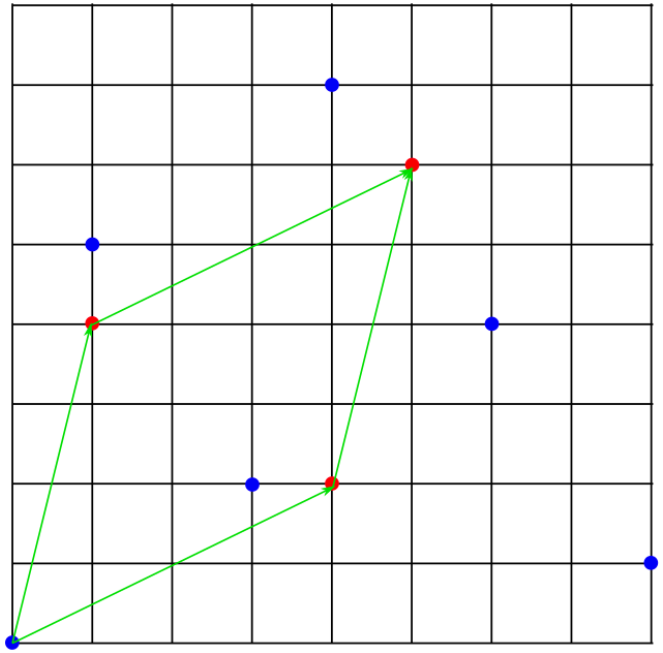
What is post-quantum cryptography ?

Lattices (Kyber)



Closest vector problem:
Given a matrix **A**
and a vector **b** = **x** + **Ay**
Find **y** such that **x** is of the small norm

Codes (HQC)



Syndrome decoding problem:
Given a generator matrix **A**
and a noisy codeword **b** = **x** + **Ay**, with x of low weight
Find **y** such that **x** of small weight

The same, but different, but the same

Kyber:

KeyGen:

$\mathbf{A} \leftarrow R^{k \cdot k}$

SK = $(\mathbf{e}, \mathbf{s}) \leftarrow R^{k}$ from a binomial distribution

PK = $(\mathbf{A}, \mathbf{t} = \mathbf{A}\mathbf{s} + \mathbf{e})$

Encrypt:

$\mathbf{r}, \mathbf{e}_1, \mathbf{e}_2 \leftarrow R^{k}$ from a binomial distribution

CT = $(\mathbf{u}, \mathbf{v}) = (\mathbf{A}\mathbf{r} + \mathbf{e}_1$, Decompress(m) + $\mathbf{t}\mathbf{r} + \mathbf{e}_2)$

Decrypt:

m' = Compress($\mathbf{v} - \mathbf{u}\mathbf{s}$)

= Compress(D(m) + $\mathbf{A}\mathbf{s}\mathbf{r} + \mathbf{e}\mathbf{r} + \mathbf{e}_2 + \mathbf{A}\mathbf{s}\mathbf{r} + \mathbf{s}\mathbf{e}_{1'}$)

= Compress(D(m) + $\mathbf{e}\mathbf{r} + \mathbf{e}_2 + \mathbf{s}\mathbf{e}_{1'}$)

= m with overwhelming probability

HQC:

KeyGen:

$\mathbf{h} \leftarrow R$

SK = $(\mathbf{x}, \mathbf{y}) \leftarrow R_{\omega}^{2}$

PK = $(\mathbf{h}, \mathbf{s} = \mathbf{h} \cdot \mathbf{y} + \mathbf{x})$

Encrypt:

$\mathbf{r}_1, \mathbf{r}_2, \mathbf{e} \leftarrow R_{\omega}^{2}$

CT = $(\mathbf{u}, \mathbf{v}) = (\mathbf{h} \cdot \mathbf{r}_2 + \mathbf{r}_1$, mG + $\mathbf{s} \cdot \mathbf{r}_2 + \mathbf{e})$

Decrypt:

m' = Decode($\mathbf{v} - \mathbf{u} \cdot \mathbf{y}$)

= Decode(mG + $\mathbf{y} \cdot \mathbf{h} \cdot \mathbf{r}_2 + \mathbf{x} \cdot \mathbf{r}_2 + \mathbf{e} + \mathbf{y} \cdot \mathbf{h} \cdot \mathbf{r}_2 + \mathbf{y} \cdot \mathbf{r}_1$)

= Decode(mG + $\mathbf{x} \cdot \mathbf{r}_2 + \mathbf{e} + \mathbf{y} \cdot \mathbf{r}_1$)

= m with overwhelming probability

HQC has worse performance than Kyber, but has a less intensive history of optimization

Contents

1. Why is HQC hard to improve ?
2. How do we propose to bypass these limitations to improve HQC ?
3. Why improving HQC can sometimes be dangerous ?

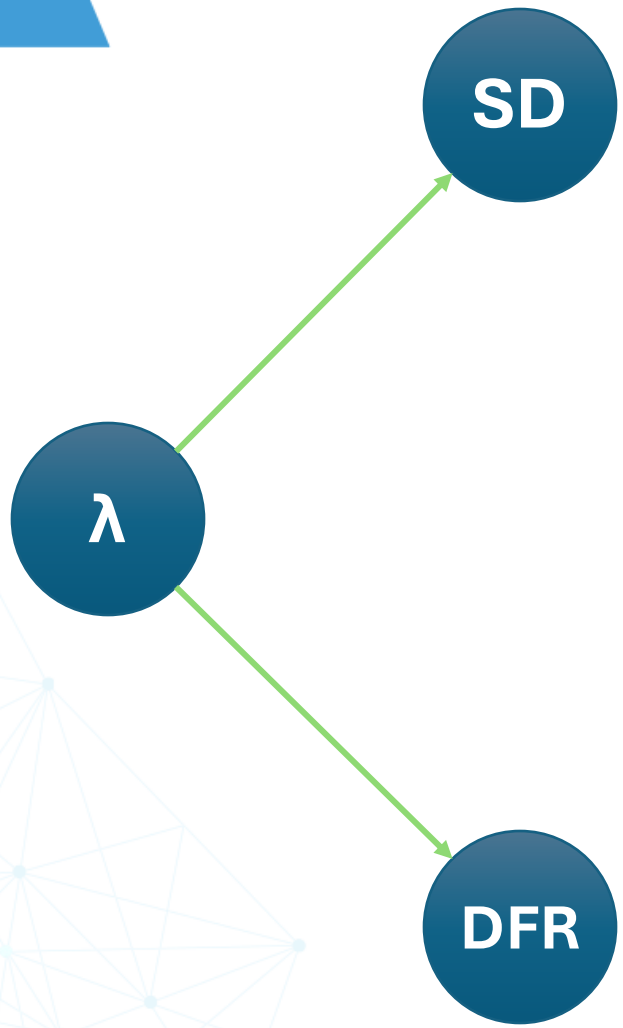
N : the golden parameter

PK size	$O(N)$	3024 bytes
SK size	$O(N)$	3064 bytes
CT size	$O(2 \cdot N)$	6017 bytes
KeyGen	mult	$2^{17.60}$ cycles
Encrypt	2·mult + encode	$2^{18.32}$ cycles
Decrypt	3·mult + encode + decode	$2^{19.24}$ cycles

mult	$O(N^{1.6})$
encode	$O(N)$ (~2% of mult)
decode	$O(N)$ (~10% of mult)

A web of parameters interactions

A web of parameters interactions



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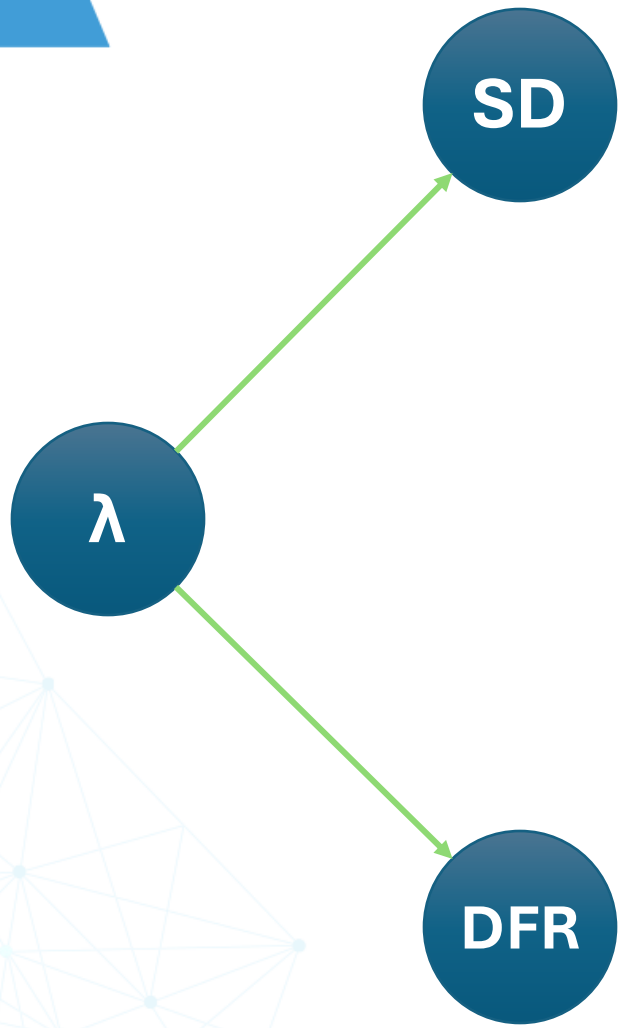


Weight of low-weigth polynomials

ω_x ω_y ω_{r1} ω_{r2} ω_e

Size of polynomials

N

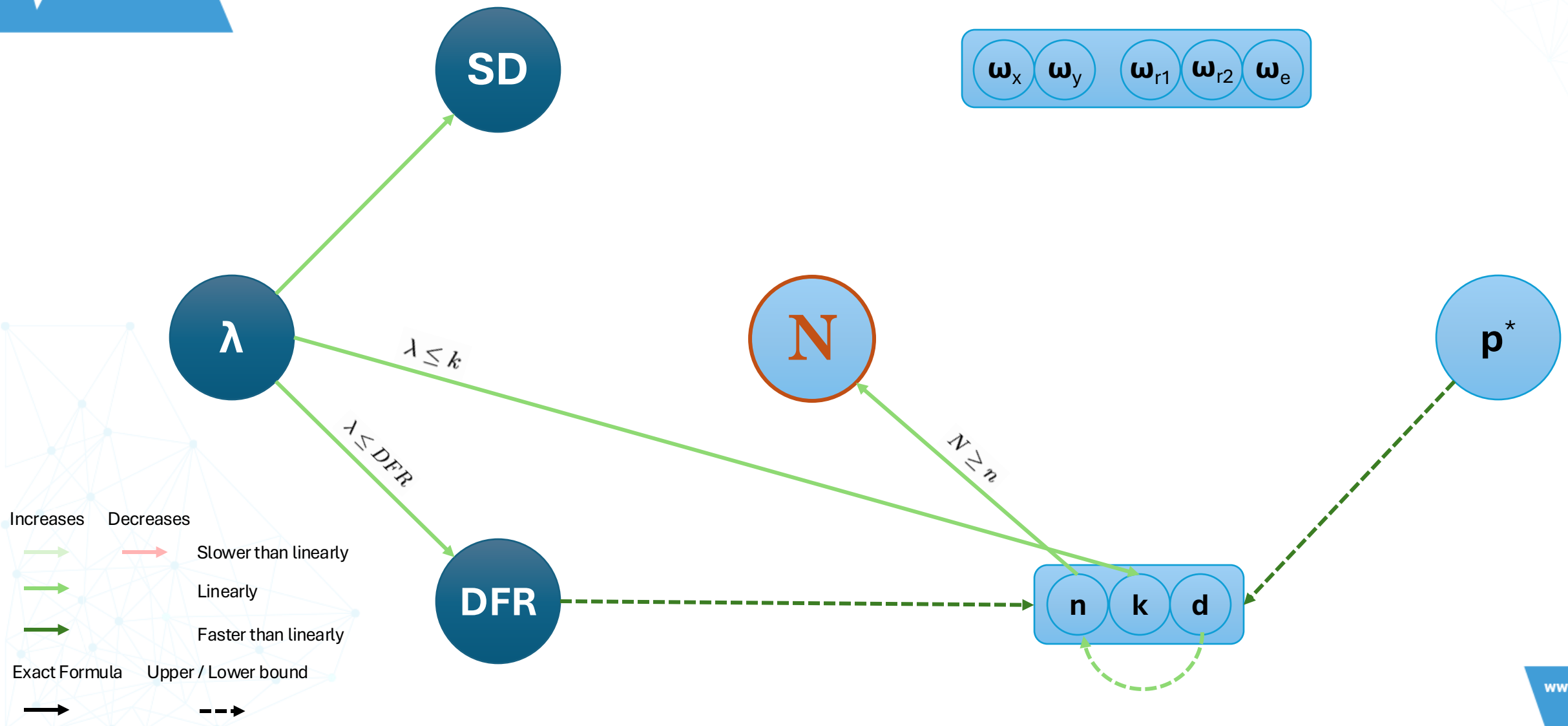
Remaining error rate before decoding

p^*

Error Correcting Code

n k d

A web of parameters interactions



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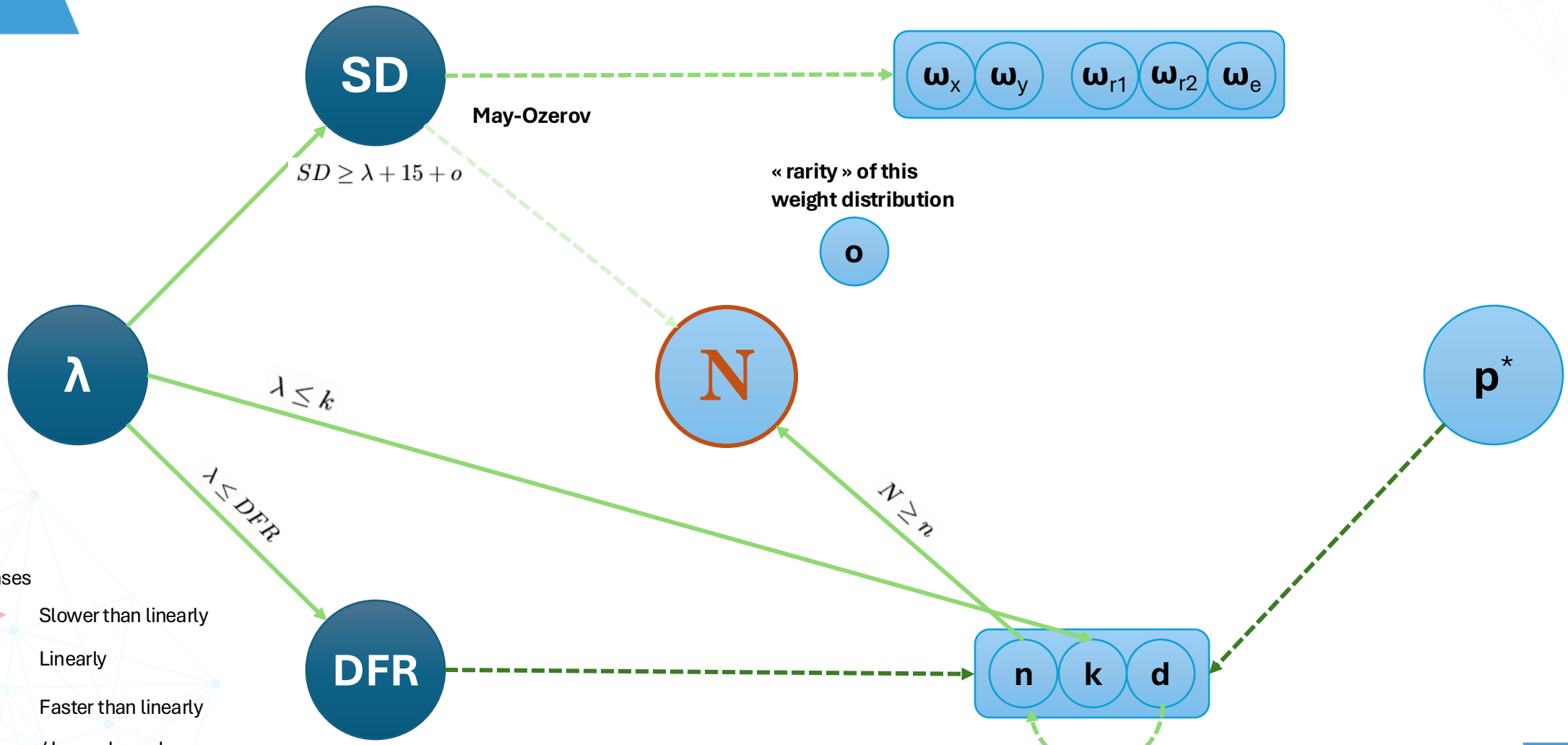
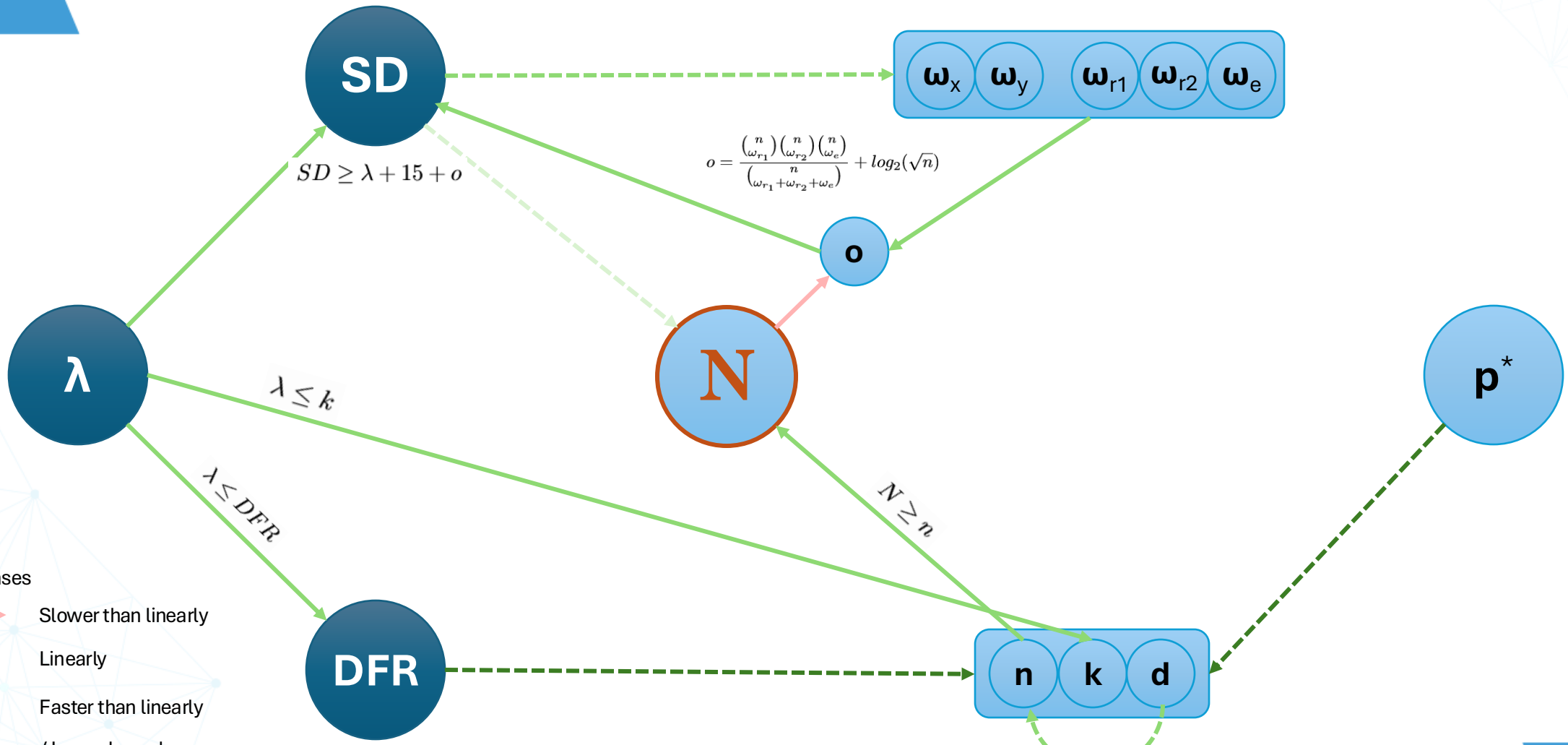


A web of parameters interactions



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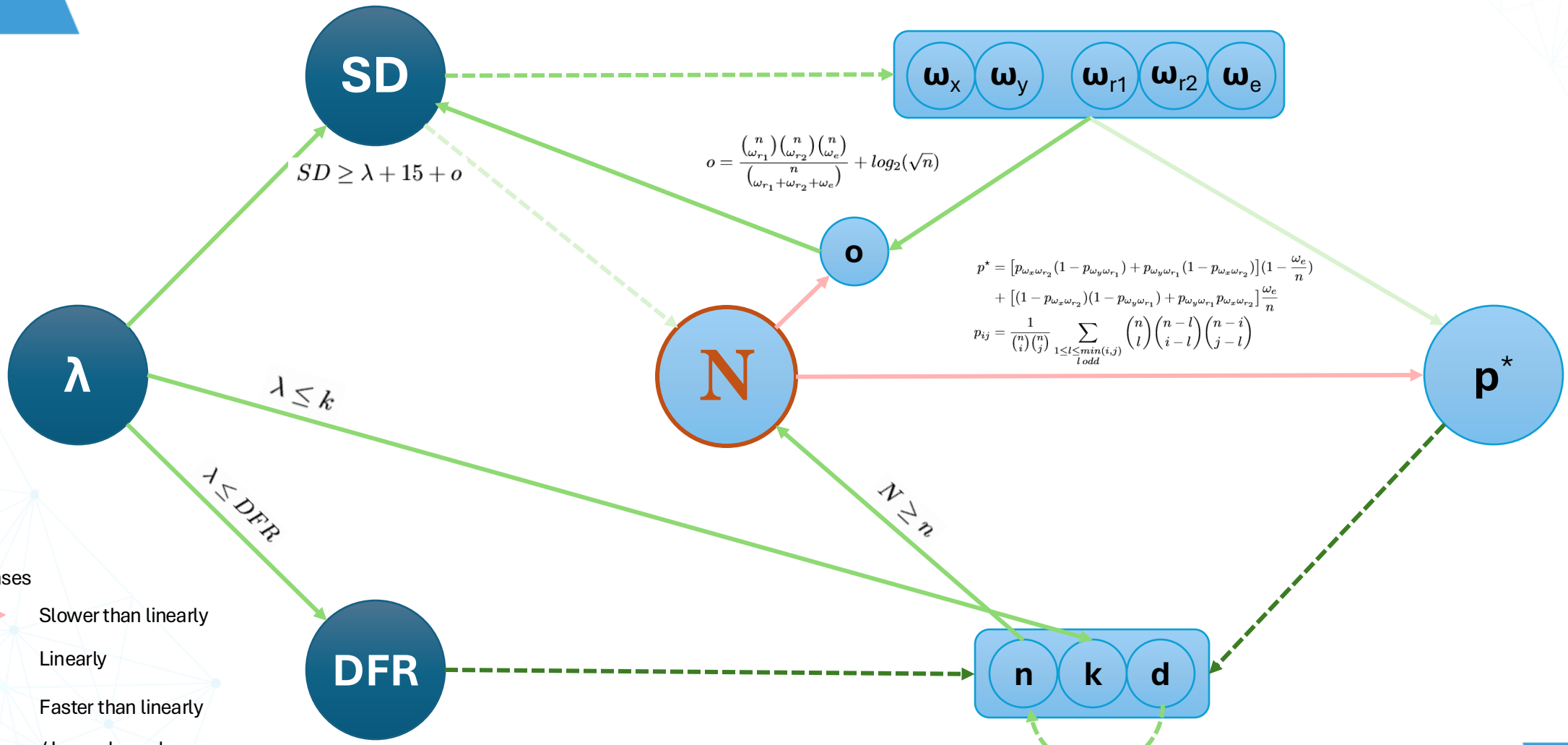
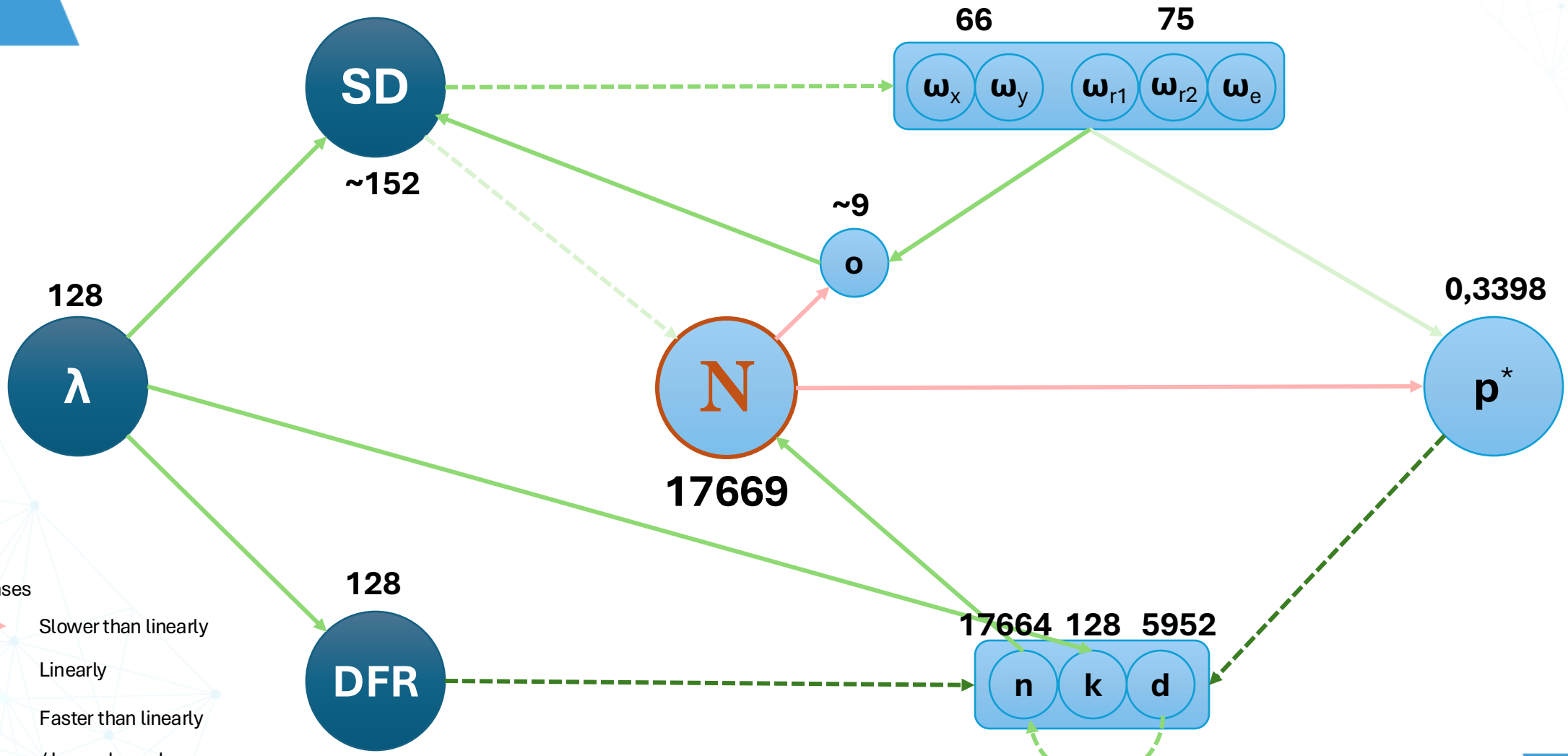
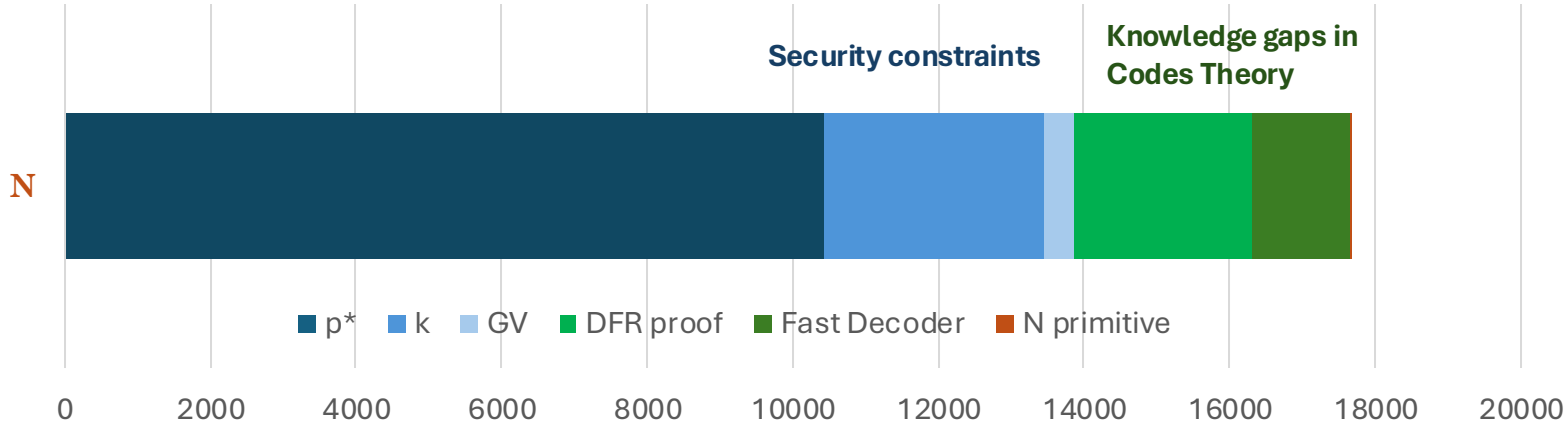


A web of parameters interactions

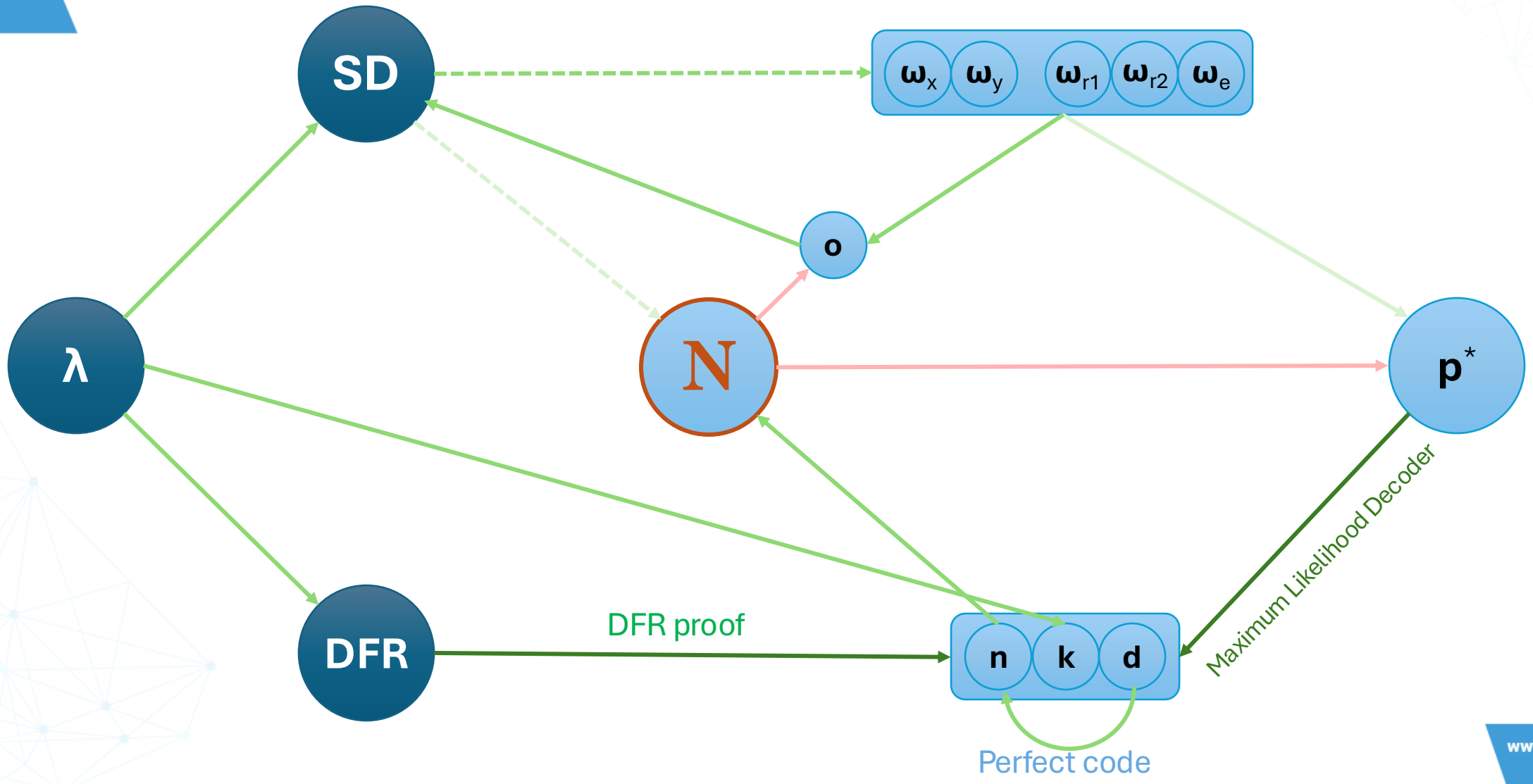


Why is N this big ?

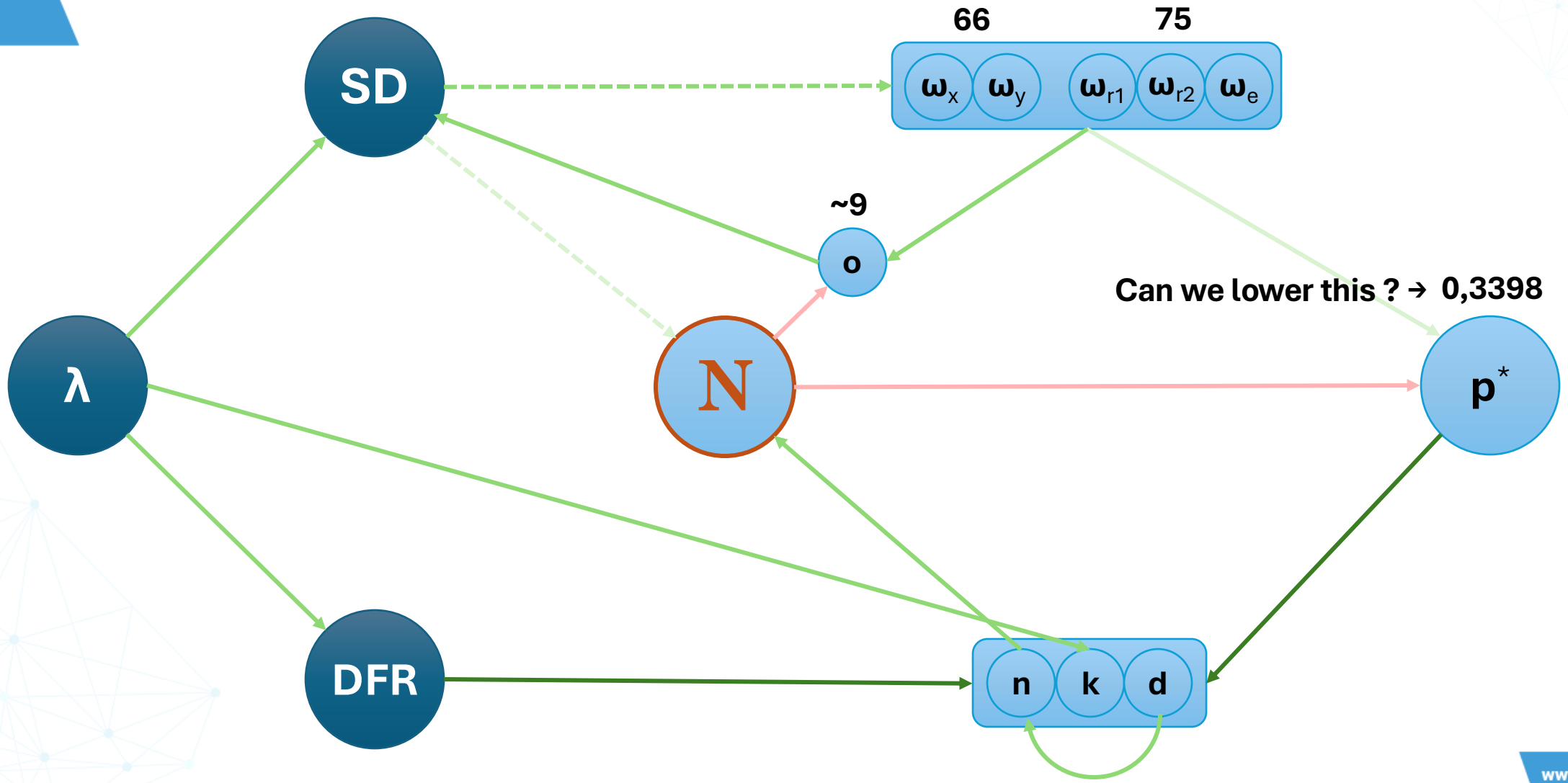


1. For this noise level, even for a **1 bit** message, no smaller code can have a correct DFR
2. Existence bound : there is no code for a **k bit** message below this size
3. GV bound : **codes** are proved to **exist** above this size
4. DFR proof : a tighter **bound**, or another code family can be found
5. Decoder : could be **slower** to correct more errors

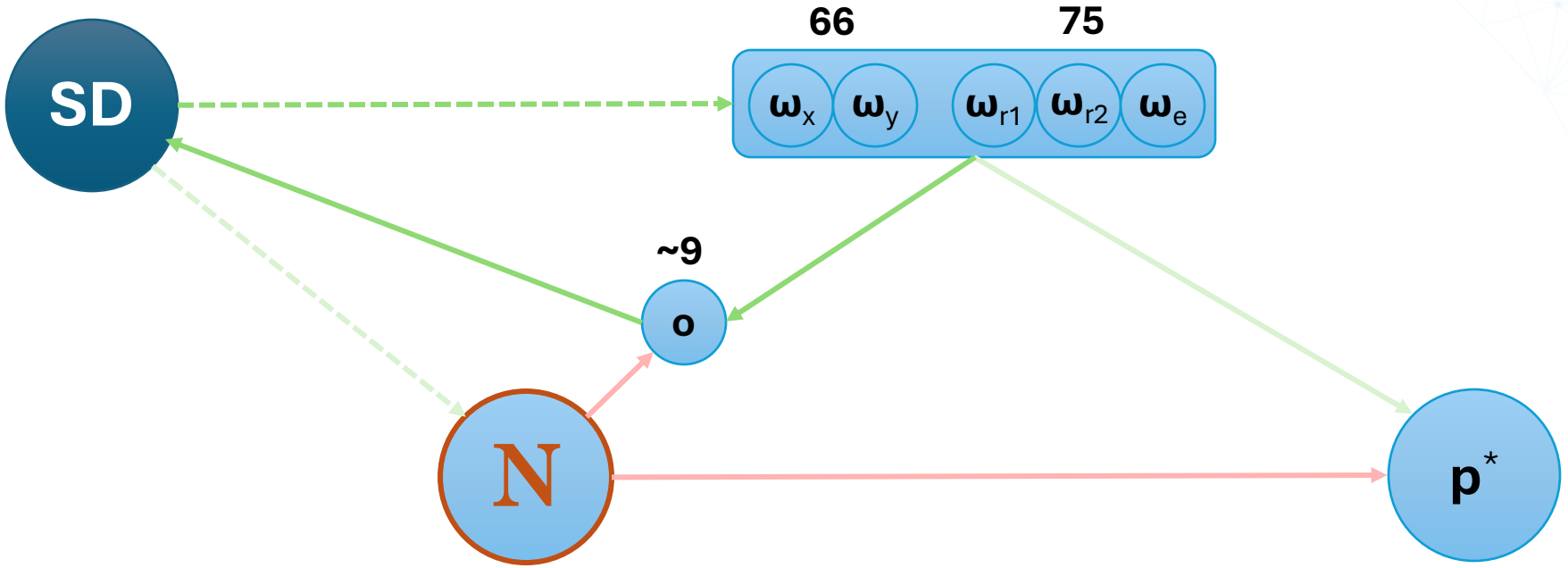
A focus on the code



« Make your requirements less dumb »



Reducing residual errors



$$p^\star = \left[p_{\omega_x\omega_{r_2}}(1-p_{\omega_y\omega_{r_1}}) + p_{\omega_y\omega_{r_1}}(1-p_{\omega_x\omega_{r_2}})\right](1-\frac{\omega_e}{n}) + \left[(1-p_{\omega_x\omega_{r_2}})(1-p_{\omega_y\omega_{r_1}}) + p_{\omega_y\omega_{r_1}}p_{\omega_x\omega_{r_2}}\right]\frac{\omega_e}{n}$$

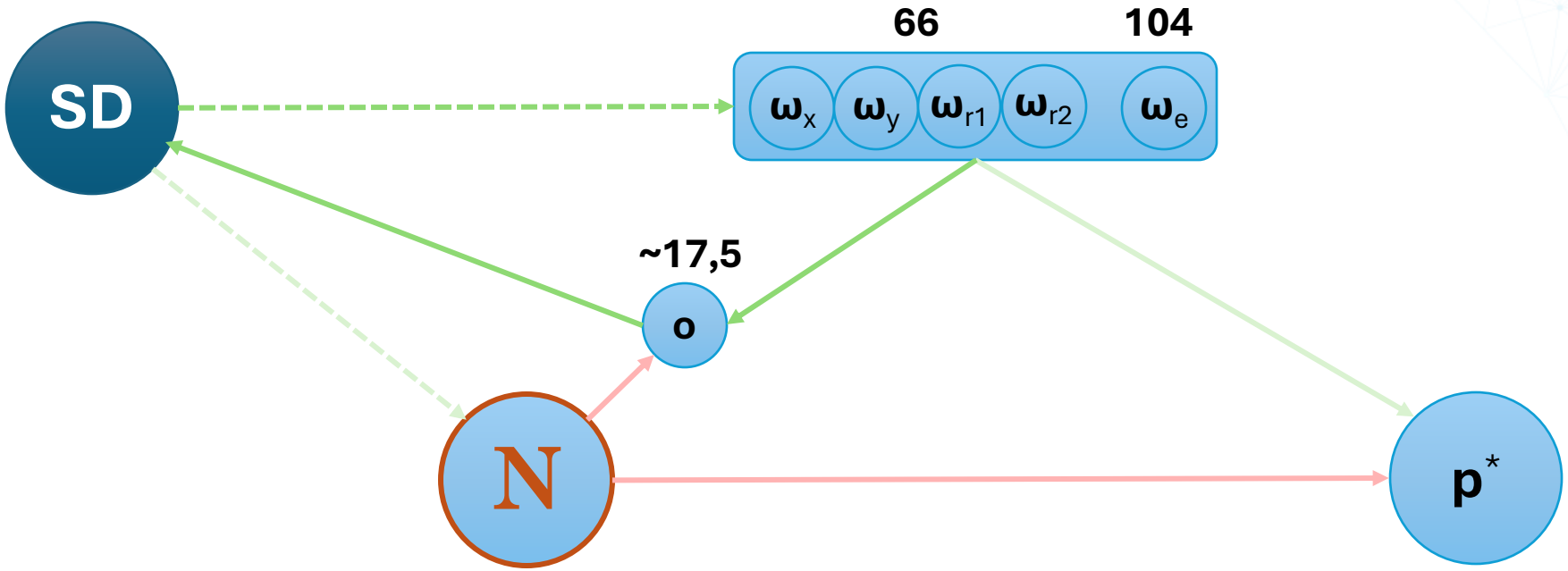
$$p_{ij} = \frac{1}{\binom{n}{i}\binom{n}{j}} \sum_{\substack{1\le l\le min(i,j)\\ l\ odd}} \binom{n}{l}\binom{n-l}{i-l}\binom{n-i}{j-l}$$

ω_x ω_y ω_{r1} ω_{r2} all have the same **quadratic** impact

ω_e has a smaller, **linear** impact

Noise can be moved in ω_e **,** but the distribution becomes rarer (**o** increases)
→ the noise is not moved **1:1,** but **p*** is still **reduced**

Reducing residual errors



$$p^\star = \left[p_{\omega_x\omega_{r_2}}(1-p_{\omega_y\omega_{r_1}}) + p_{\omega_y\omega_{r_1}}(1-p_{\omega_x\omega_{r_2}})\right](1-\frac{\omega_e}{n}) + \left[(1-p_{\omega_x\omega_{r_2}})(1-p_{\omega_y\omega_{r_1}}) + p_{\omega_y\omega_{r_1}}p_{\omega_x\omega_{r_2}}\right]\frac{\omega_e}{n}$$

$$p_{ij} = \frac{1}{\binom{n}{i}\binom{n}{j}} \sum_{\substack{1\le l\le min(i,j)\\ l\ odd}} \binom{n}{l}\binom{n-l}{i-l}\binom{n-i}{j-l}$$

ω_x ω_y ω_{r1} ω_{r2} all have the same **quadratic** impact

ω_e has a smaller, **linear** impact

Noise can be moved in ω_e **,** but the distribution becomes rarer (**o** increases)
→ the noise is not moved **1:1,** but **p*** is still **reduced**

With this change we can reduce N by **7%**

« Make your requirements less dumb » - again

Do we really want to reduce N ?

Decode takes a size n input
u has to be of size N, but **v** only of size n.

Right now the two are very close,
but they do not have to be…

As **increasing** N reduces errors,
smaller codes can be found,
therefore reducing ciphertext size (N + n).

HQC:

KeyGen:

$\mathbf{h} \leftarrow R$

$SK = (\mathbf{x}, \mathbf{y}) \leftarrow R_\omega^2$

$PK = (\mathbf{h}, \mathbf{s} = \mathbf{h}\cdot\mathbf{y} + \mathbf{x})$

Encrypt:

$\mathbf{r_1}, \mathbf{r_2}, \mathbf{e} \leftarrow R_w^2$

$CT = (\mathbf{u}, \mathbf{v}) = (\mathbf{h}\cdot\mathbf{r_2} + \mathbf{r_1}, mG + \mathbf{s}\cdot\mathbf{r_2} + \mathbf{e})$

Decrypt:

$m' = Decode(\mathbf{v} - \mathbf{u}\cdot\mathbf{y})$

$= Decode(mG + \mathbf{y}\cdot\mathbf{h}\cdot\mathbf{r_2} + \mathbf{x}\cdot\mathbf{r_2} + \mathbf{e} + \mathbf{y}\cdot\mathbf{h}\cdot\mathbf{r_2} + \mathbf{y}\cdot\mathbf{r_1})$

$= Decode(mG + \mathbf{x}\cdot\mathbf{r_2} + \mathbf{e} + \mathbf{y}\cdot\mathbf{r_1})$

$= m$ with overwhelming probability

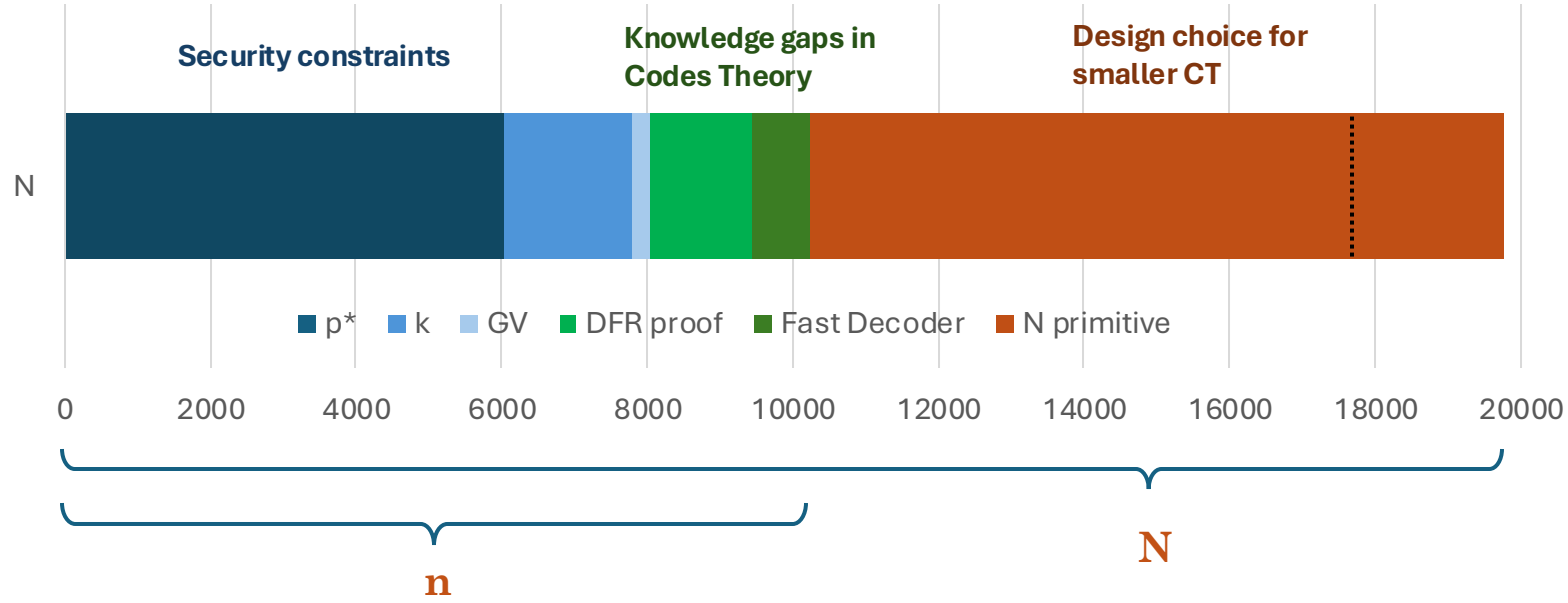
N and n, the important parameters

PK size	O(**N**)	+ 11%
SK size	O(**N**)	+ 11%
CT size	O(**N+n**)	- 15%
KeyGen	multNN	+ 18%
Encrypt	multNN + multNn + encode	> - 8%
Decrypt	multNN + 2·multNn + encode + decode	> - 18%
multNN	O(**N**$^{1.6}$)	+18%
multNn	O((**N**·**n**)$^{?}$)	> - 35%*
encode	O(**n**)	~ - 42%
decode	O(**n**)	~ - 25%

This trade-off is better for reused keys

*Polynomial multiplication is already very optimized (Toom cook + Karatsuba) Speed-up will likely be less than schoolbook

Why are N and n this big ?



Noise placement has been optimized

Minimal CT has been found, but CT/SK trade-offs are possible

Improvements on Code Theory are now less impactful

Improvements on implementation are crucial, but may lead to weaknesses…

Attacks on HQC decryption

Decode:

```
// Retrieve x, y, pk from secret key
hqc_secret_key_from_string(x, y, sigma, pk, sk);

// Compute v - u.y
vect_resize(tmp1, PARAM_N, v, PARAM_N1N2);
vect_mul(tmp2, y, u);
vect_add(tmp2, tmp1, tmp2, VEC_N_SIZE_64);

// Compute m by decoding v - u.y
code_decode(m, tmp2);
```

```
void code_decode(uint64_t *m, const uint64_t *em) {
    uint64_t tmp[VEC_N1_SIZE_64] = {0};

    reed_muller_decode(tmp, em);
    reed_solomon_decode(m, tmp);
}
```

```
void reed_muller_decode(uint64_t *msg, const uint64_t *cdw) {
    uint8_t *message_array = (uint8_t *) msg;
    codeword *codeArray = (codeword *) cdw;
    expandedCodeword expanded;
    for (size_t i = 0; i < VEC_N1_SIZE_BYTES; i++) {
        // collect the codewords
        expand_and_sum(&expanded, &codeArray[i * MULTIPLICITY]);
        // apply hadamard transform
        expandedCodeword transform;
        hadamard(&expanded, &transform);
        // fix the first entry to get the half Hadamard transform
        transform[0] -= 64 * MULTIPLICITY;
        // finish the decoding
        message_array[i] = find_peaks(&transform);
    }
```

```
void expand_and_sum(expandedCodeword *dest, codeword src[]) {
    // start with the first copy
    for (int32_t part = 0; part < 4; part++) {
        for (int32_t bit = 0; bit < 32; bit++) {
            (*dest)[part * 32 + bit] = src[0].u32[part] >> bit & 1;
        }
    }
    // sum the rest of the copies
    for (int32_t copy = 1; copy < MULTIPLICITY; copy++) {
        for (int32_t part = 0; part < 4; part++) {
            for (int32_t bit = 0; bit < 32; bit++) {
                (*dest)[part * 32 + bit] += src[copy].u32[part] >> bit & 1;
            }
        }
    }
}
```

The input of `expand_and_sum` is $\mathbf{v} - \mathbf{u} \cdot \mathbf{y}$

v and **u** are known, as parts of the ciphertext

They can be used to retrieve the secret key **y**

Attacks on HQC decryption

Expand_and_sum:

```
void expand_and_sum(expandedCodeword *dest, codeword src[]) {
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            for (int32_t bit = 0; bit < 32; bit++) {
                (*dest)[part * 32 + bit] += src[copy].u32[part] >> bit & 1;
            }
        }
    }
}
```
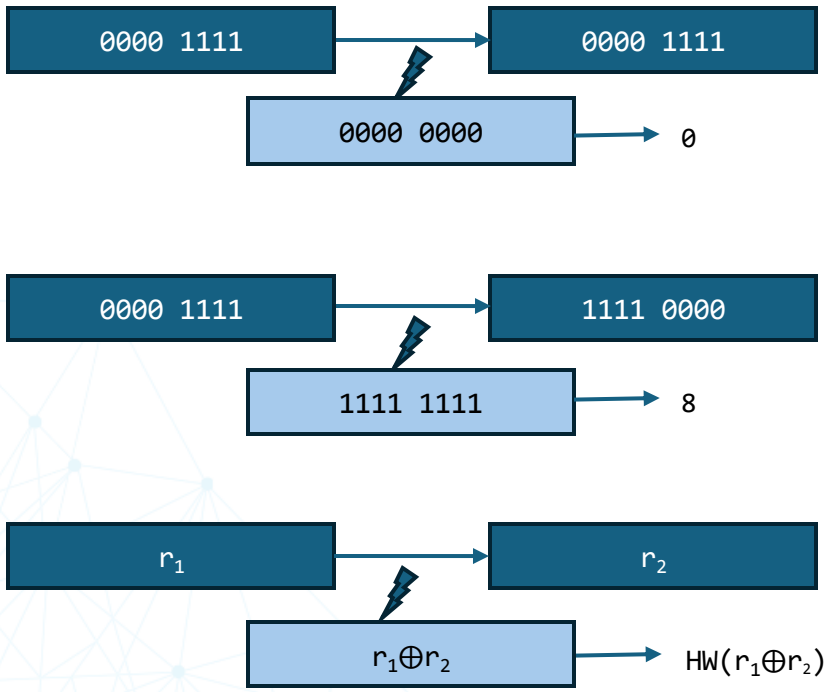
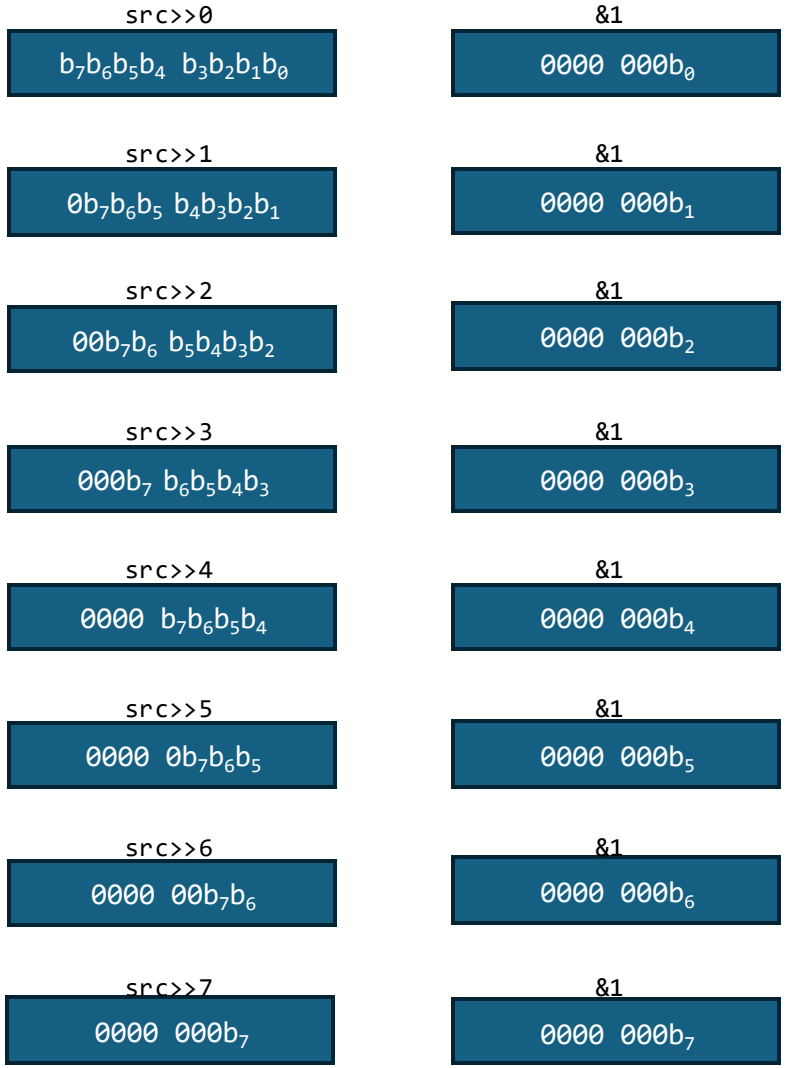

src>>0	&1
$b_7b_6b_5b_4\ b_3b_2b_1b_0$	0000 000b_0
src>>1	&1
$0b_7b_6b_5\ b_4b_3b_2b_1$	0000 000b_1
src>>2	&1
$00b_7b_6\ b_5b_4b_3b_2$	0000 000b_2
src>>3	&1
$000b_7\ b_6b_5b_4b_3$	0000 000b_3
src>>4	&1
0000 $b_7b_6b_5b_4$	0000 000b_4
src>>5	&1
0000 $0b_7b_6b_5$	0000 000b_5
src>>6	&1
0000 $00b_7b_6$	0000 000b_6
src>>7	&1
0000 000b_7	0000 000b_7

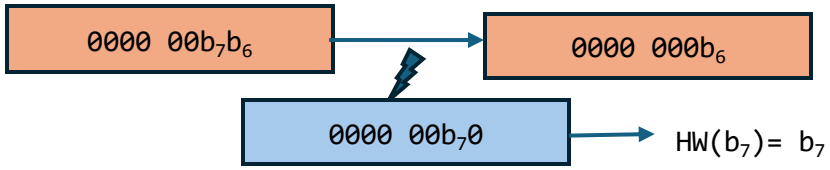
Side Channel Attacks – Instruction Set

Leakage in Hamming distance model



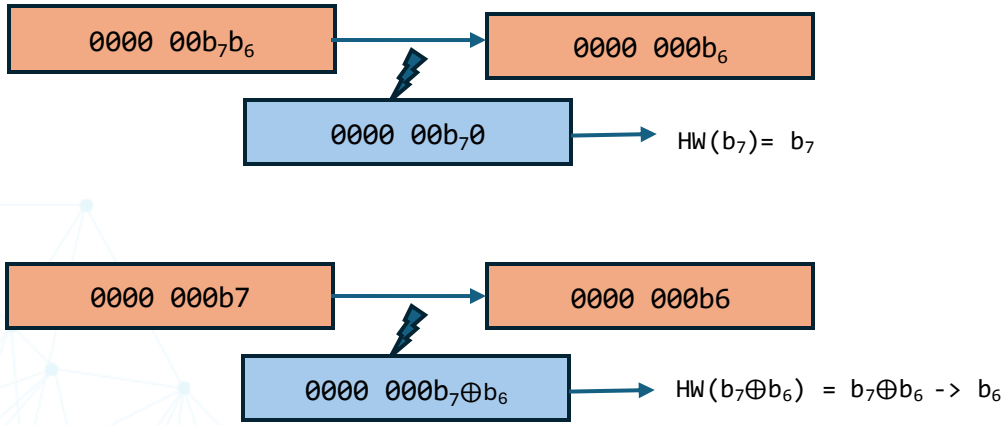


Side Channel Attacks – Instruction Set



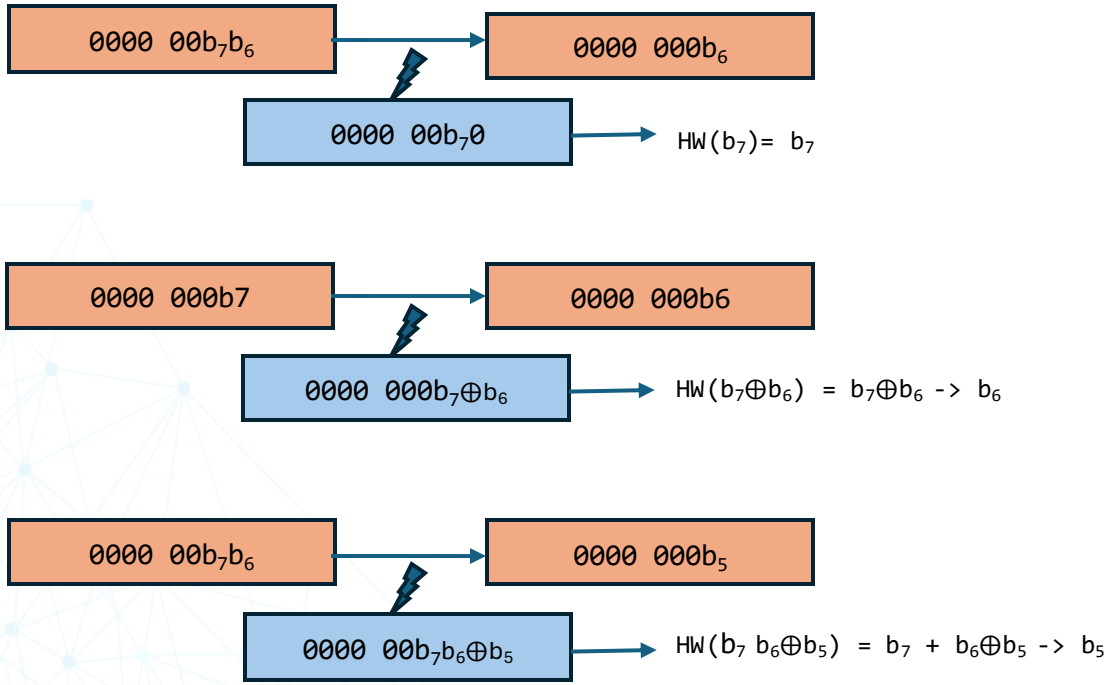
src>>0	&1
$b_7b_6b_5b_4\ b_3b_2b_1b_0$	0000 000b_0
src>>1	&1
$0b_7b_6b_5\ b_4b_3b_2b_1$	0000 000b_1
src>>2	&1
$00b_7b_6\ b_5b_4b_3b_2$	0000 000b_2
src>>3	&1
$000b_7\ b_6b_5b_4b_3$	0000 000b_3
src>>4	&1
0000 $b_7b_6b_5b_4$	0000 000b_4
src>>5	&1
0000 $0b_7b_6b_5$	0000 000b_5
src>>6	&1
0000 $00b_7b_6$	0000 000b_6
src>>7	&1
0000 000b_7	0000 000b_7

Side Channel Attacks – Instruction Set

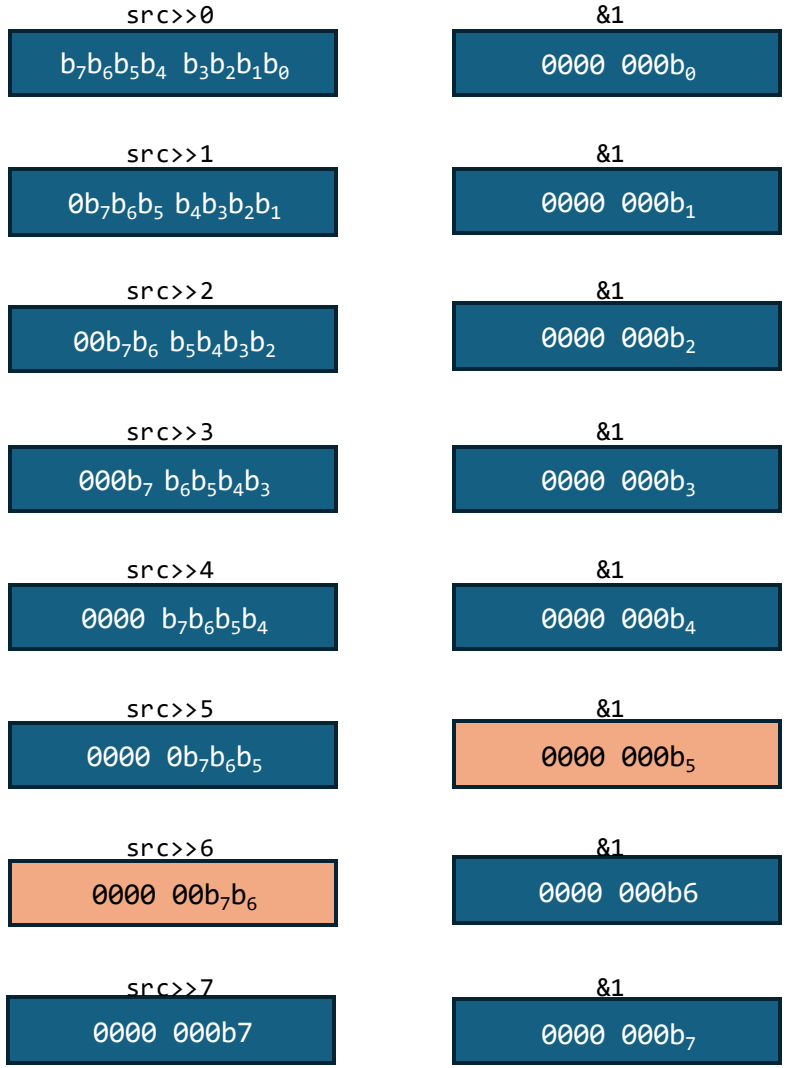


src>>0	&1
$b_7b_6b_5b_4\ b_3b_2b_1b_0$	0000 000b_0
src>>1	&1
$0b_7b_6b_5\ b_4b_3b_2b_1$	0000 000b_1
src>>2	&1
$00b_7b_6\ b_5b_4b_3b_2$	0000 000b_2
src>>3	&1
$000b_7\ b_6b_5b_4b_3$	0000 000b_3
src>>4	&1
0000 $b_7b_6b_5b_4$	0000 000b_4
src>>5	&1
0000 $0b_7b_6b_5$	0000 000b_5
src>>6	&1
0000 $00b_7b_6$	0000 000b_6
src>>7	&1
0000 000b_7	0000 000b_7

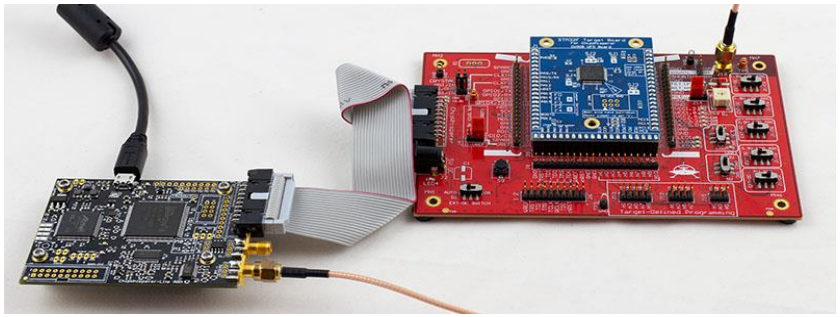
Side Channel Attacks – Instruction Set

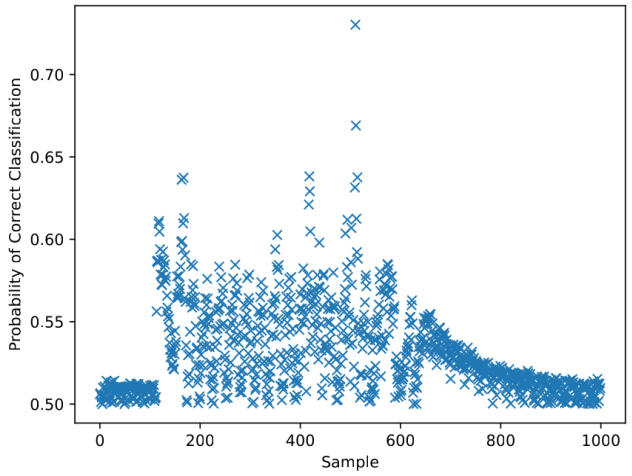


Repeat to find all bits



Side Channel Attacks – Micro Architecture





(b) Probability of correct classification of bit_{10} for each sample

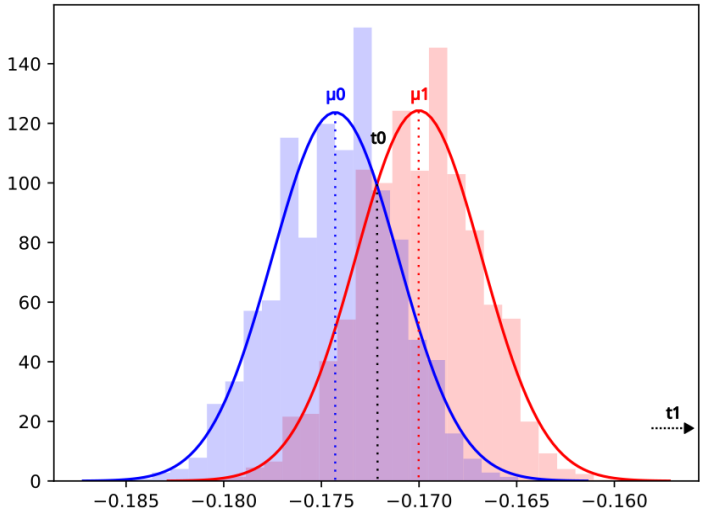
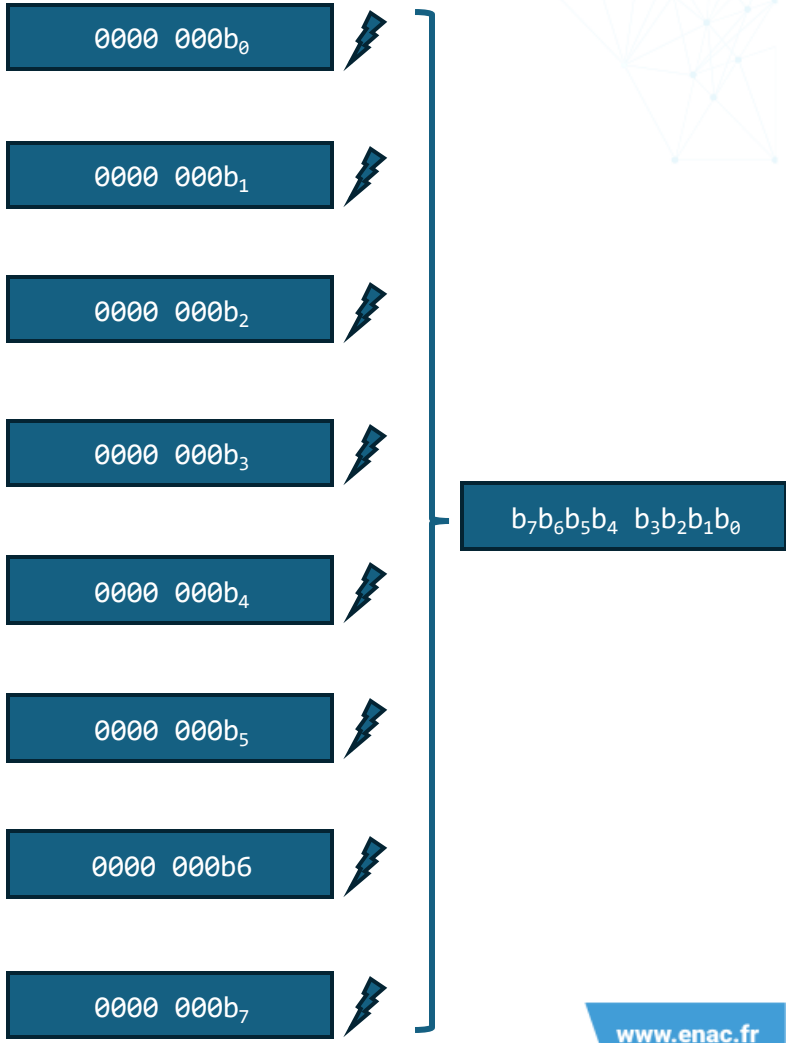


Fig. 7: Samples distributions for 0 and 1 on the POI of bit_0



Side Channel Attacks – Micro Architecture

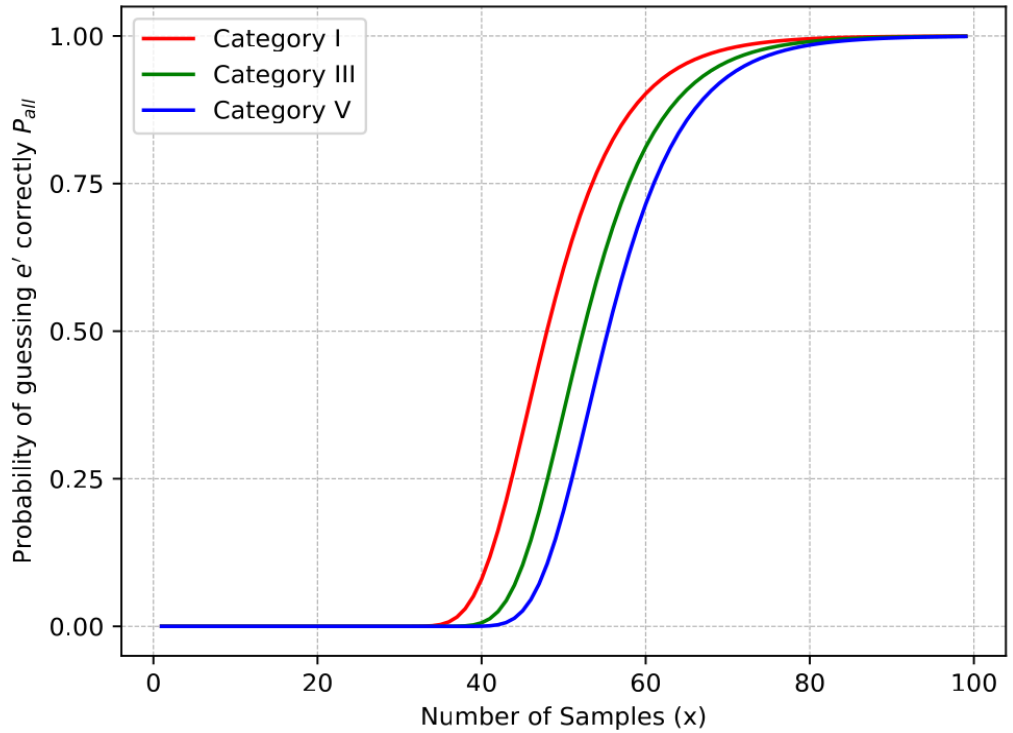
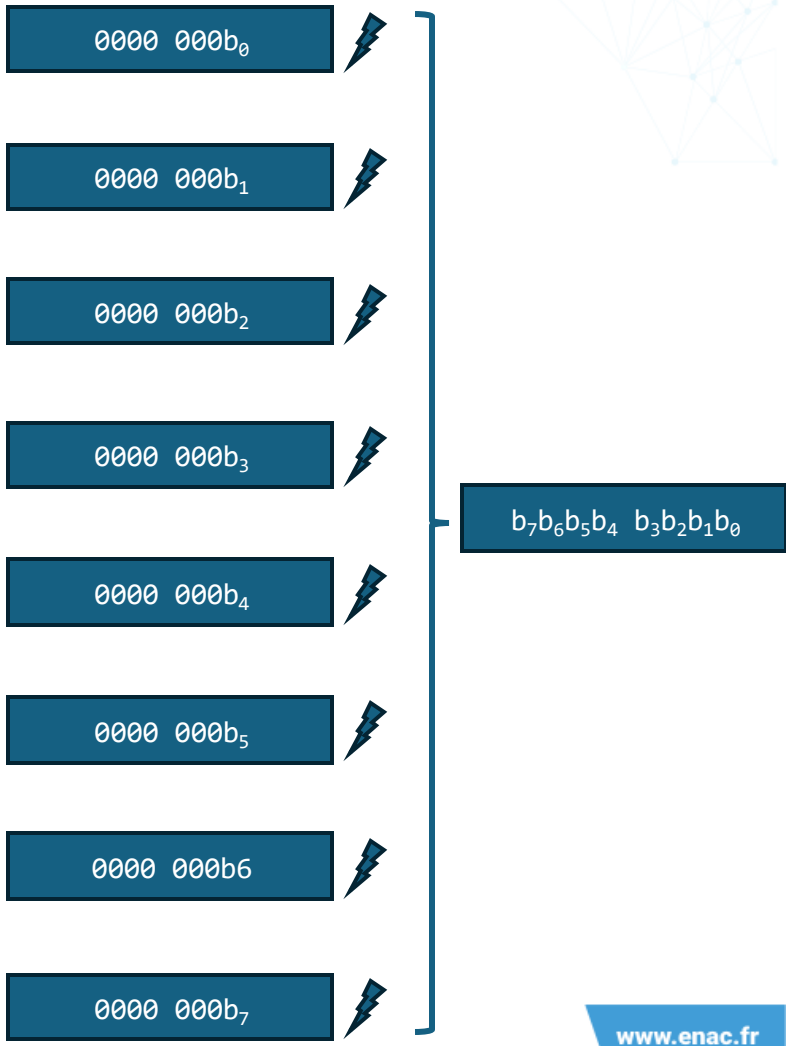


Fig. 9: Probabilities that $\mathbf{v} - \mathbf{u} \cdot \mathbf{y}$ is guessed correctly according to the number of traces and security category.



Attacks on HQC decryption

Some possible countermeasures

1. Change the decoder to a Minimum Distance Decoder to **avoid `expand_and_sum`** (5.3% slowdown)

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1. Change the decoder to a Minimum Distance Decoder to **avoid `expand_and_sum`** (5.3% slowdown)

2. **Add noise** to the input (0.14% slowdown)

Decrypt:
m' = Decode(**v** - **u**·**y**)

Replace by
e'' ← $R_{\omega'', seed}$(**u**, **v**)
m' = Decode(**v** - **u**·**y** + **e''**)

Leakage still happens, but too many possible **e''**, no way to bruteforce **y**
Has an impact on DFR, but negligible

Attacks on HQC decryption

Some possible countermeasures

1. Change the decoder to a Minimum Distance Decoder to **avoid `expand_and_sum`** (5.3% slowdown)

2. Add **noise** to the input (0.14% slowdown)

3. **Mask the codeword** (1.14% slowdown)

Decrypt:
$m' = \text{Decode}(\mathbf{v} - \mathbf{u}\cdot\mathbf{y})$

Replace by
$m'' \leftarrow \{0,1\}^k$
$m' = \text{Decode}(\mathbf{v} - \mathbf{u}\cdot\mathbf{y} + m''G) \oplus m''$

Leakage still happens but computing $\mathbf{u}^{-1}(\mathbf{v} - (\mathbf{v} - \mathbf{u}\cdot\mathbf{y} + m''G))$ does not return $\mathbf{y}$

Conclusion

1. **Why is HQC hard to improve ?**
   - ➔ Interactions between many variables which cause $\mathbf{N}$ to increase
   - ➔ Based on state-of-the-art Error Correcting Codes

2. **How do we propose to bypass these limitations to improve HQC ?**
   - ➔ Move some of the weight of $\mathbf{r}_1$ and $\mathbf{r}_2$ to $\mathbf{e}$ (at a cost)
   - ➔ Still the residual noise and $\mathbf{N}$ are lower
   - ➔ Also, a trade-off is to increase $\mathbf{N}$ to decrease CT, Encrypt and Decrypt

3. **Why improving HQC can sometimes be dangerous ?**
   - ➔ The most optimized decryption leaks the key
   - ➔ Countermeasures exist but requires slowdown

Adapting Cryptographic Tools to a Post-Quantum Context

Or how to improve HQC

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